

Introduction to Actuarial Mathematics

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Key concepts, tools, and applications for students beginning their actuarial studies.

1. What Is Actuarial Mathematics?

Aspect	Description
Definition	The quantitative discipline that applies probability, statistics, and financial theory to assess and manage risk in insurance, pensions, and other financial domains.
Goal	To value uncertain future cash flows, design fair contracts, and ensure the financial stability of institutions that assume risk.
Core Fields	- Life insurance & annuities - General insurance (property & casualty) - Pensions & employee benefits - Health insurance - Enterprise risk management

2. Fundamental Mathematical Foundations

Topic	Why It Matters	Typical Use in Actuarial Work
Probability Theory	Models randomness of lifetimes, claim occurrences, etc.	Survival probabilities, claim count distributions
Statistics & Estimation	Derives parameters from data (mortality, lapse, claim severity).	Maximum likelihood, method of moments, Bayesian updating
Financial Mathematics	Converts future uncertain cash flows to present values.	Discounting, accumulation, interest rate models
Stochastic Processes	Describes evolution of variables over time (e.g., interest rates, claim counts).	Poisson processes, Markov chains, Brownian motion

3. Core Actuarial Models

3.1 Life Table & Survival Functions

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Symbol	Meaning
(ℓ_x)	Number of lives alive at exact age (x) (from a radix, usually 100,000).
$(d_x = \ell_x - \ell_{x+1})$	Deaths between ages (x) and (x+1).
$(q_x = d_x / \ell_x)$	Probability of dying within one year at age (x).
$(p_x = 1 - q_x)$	Probability of surviving one year at age (x).
$({}_t p_x = \frac{\ell_{x+t}}{\ell_x})$	Probability of surviving (t) years from age (x).
$({}_t q_x = 1 - {}_t p_x)$	Probability of dying within (t) years from age (x).

Key relationships

$$[{}_t p_x = \prod_{k=0}^{t-1} p_{x+k}, \quad {}_t q_x = 1 - {}_t p_x]$$

3.2 Present Value Random Variables

Notation	Interpretation
(Z)	Random present value of a benefit (e.g., death benefit).
$(v = (1+i)^{-1})$	Discount factor for one period at interest rate (i).
$(A_x = \mathbb{E}[Z])$	Actuarial present value (APV) of a benefit for a life aged (x).
$(\text{Var}(Z))$	Measure of risk (used in capital requirement calculations).

Example – Whole life insurance

$$[A_x = \sum_{k=0}^{\infty} v^{k+1} {}_k p_x q_{x+k}]$$

3.3 Annuities

Symbol	Meaning
$(\overline{a}_n)$	