

Introduction to Probability

Introduction to Probability – Study Notes

1. What Is Probability?

Concept	Description
Probability	A numerical measure of how likely an event is to occur, ranging from 0 (impossible) to 1 (certain).
Experiment	Any process that generates outcomes (e.g., rolling a die, drawing a card).
Sample Space (S)	The set of all possible outcomes of an experiment.
Event	A subset of the sample space (one or more outcomes).

Example:

- Experiment: Toss a fair coin twice.
 - Sample space: $(S = \{HH, HT, TH, TT\})$.
 - Event A = “exactly one head” = $(\{HT, TH\})$.
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2. Classical (Theoretical) Probability

For a **finite, equally likely** sample space:

$$[P(A) = \frac{\text{Number of outcomes in } A}{\text{Number of outcomes in } S}]$$

Example: Probability of drawing an Ace from a standard 52-card deck

$$[P(\text{Ace}) = \frac{4}{52} = \frac{1}{13}]$$

3. Axiomatic Foundations (Kolmogorov)

- Non-negativity:** $(P(A) \geq 0)$ for any event (A).
- Normalization:** $(P(S) = 1)$.
- Additivity** (for mutually exclusive events):

$$[P(A \cup B) = P(A) + P(B) \quad \text{if } A \cap B = \varnothing]$$

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From these axioms follow useful rules:

- **Complement:** $(P(A^c) = 1 - P(A))$
- **General addition rule:**
 $[P(A \cup B) = P(A) + P(B) - P(A \cap B)]$

4. Types of Events

Type	Definition	Probability Rule
Simple	Single outcome (e.g., rolling a 3).	Direct count or formula.
Compound	Combination of outcomes (e.g., “roll an even number”).	Use addition rule.
Mutually exclusive	Cannot occur together (e.g., rolling a 2 and a 5).	$(P(A \cup B) = P(A) + P(B)).$
Independent	Occurrence of one does not affect the other.	$(P(A \cap B) = P(A)P(B)).$
Dependent	One event influences the other.	Use conditional probability.

5. Conditional Probability & Independence

Conditional Probability

Probability of (A) given that (B) has occurred:

$$[P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \text{ provided } P(B) > 0]$$

Independence

Events (A) and (B) are independent **iff**:

$$[P(A \mid B) = P(A) \quad \text{or equivalently} \quad P(A \cap B) = P(A)P(B)]$$

6. Counting Techniques

Technique	When to Use	Formula
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Multiplication Rule

Ordered selections, each step
independent

$(n_1 \times n_2 \times \dots \times n_k)$

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