

Introduction to Derivatives

Introduction to Derivatives – Study Notes

1. What Is a Derivative?

- **Definition:** A derivative measures how a function's output changes as its input changes.
 - **Geometric interpretation:** Slope of the tangent line to the graph of a function at a given point.
 - **Notation:**
 - $f'(x)$ or $\frac{df}{dx}$ – derivative of f with respect to x
 - $\frac{d}{dx}\big|_{f(x)}$ – operator form
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2. Fundamental Concepts

Concept	Description	Symbol
Limit definition	$\lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$	—
Continuity	A function must be continuous at x for the derivative to exist there.	—
Differentiability	Existence of a finite limit above; implies local linearity.	—
Tangent line	Line that just “touches” the curve; equation: $y = f(a) + f'(a)(x-a)$.	—

3. Basic Differentiation Rules

Rule	Formula	Example
Constant rule	$\frac{d}{dx}[c] = 0$	$\frac{d}{dx}[5] = 0$
Power rule	$\frac{d}{dx}[x^n] = nx^{n-1}$	$\frac{d}{dx}[x^3] = 3x^2$
Constant multiple	$\frac{d}{dx}[c \cdot f(x)] = c \cdot f'(x)$	$\frac{d}{dx}[4 \sin x] = 4 \cos x$
Sum/Difference	$\frac{d}{dx}[f \pm g] = f' \pm g'$	$\frac{d}{dx}[x^2 + \ln x] = 2x + \frac{1}{x}$
Product rule	$(fg)' = f'g + fg'$	$\frac{d}{dx}[x \sin x] = \sin x + x \cos x$

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Rule	Formula	Example
Quotient rule	$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$	$\frac{d}{dx} \left[\frac{x}{e^x} \right] = \frac{e^x - x e^x}{e^{2x}} = \frac{1-x}{e^x}$
Chain rule	$\frac{d}{dx} [f(g(x))] = f'(g(x)) \cdot g'(x)$	$\frac{d}{dx} [\sin(x^2)] = \cos(x^2) \cdot 2x$

4. Common Derivatives

Function Derivative

$(\sin x)$	$(\cos x)$
$(\cos x)$	$(-\sin x)$
$(\tan x)$	$(\sec^2 x)$
(e^x)	(e^x)