

Quantum Computing

Quantum Computing – Structured Overview

1. What Is Quantum Computing?

Classical Computing	Quantum Computing
Bits are 0 or 1	Qubits can be 0, 1, or both (superposition)
Deterministic logic gates	Quantum gates manipulate probability amplitudes
Operations scale linearly with number of bits	Certain problems scale exponentially faster due to entanglement & interference

Key Idea: Use quantum?mechanical phenomena (superposition, entanglement, interference) to process information in ways that classical bits cannot.

2. Fundamental Concepts

2.1 Qubit

- **Definition:** Two-level quantum system (e.g., electron spin, photon polarization).
- **State representation:**
[$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \text{quad } |\alpha|^2 + |\beta|^2 = 1$]
- **Bloch sphere:** Visual tool; any pure qubit state maps to a point on the sphere's surface.

2.2 Superposition

- A qubit simultaneously occupies multiple basis states.
- Enables parallel evaluation of many possibilities.

2.3 Entanglement

- Correlation between qubits such that the state of one instantly determines the state of another, regardless of distance.
- Example (Bell state):
[$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$]

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2.4 Quantum Interference

- Probability amplitudes add constructively or destructively.
- Algorithmic step: amplify correct answers, cancel wrong ones.

2.5 Measurement

- Collapses a superposed state to a classical outcome ($|0\rangle$ or $|1\rangle$) with probabilities given by the squared amplitudes.
- Measurement is destructive for the quantum information.

3. Quantum Gates & Circuits

Gate	Matrix	Effect
X (NOT)	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	Flips
H (Hadamard)	$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	Creates equal superposition
S (Phase)	$\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$	Adds a $\pi/2$ phase to
T ($\pi/8$)	$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$	Adds a $\pi/4$ phase
CNOT	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	Entangles control & target qubits
SWAP	Exchanges two qubits	Useful for routing

Circuit model:

1. **Initialize** qubits (usually to $|0\rangle$).
2. **Apply** a sequence of quantum gates (unitary operations).
3. **Measure** selected qubits to obtain classical results.

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4. Major Quantum Algorithms

| Algorithm | Problem Addressed | Speed