

Introduction to Quantum Field Theory

Introduction to Quantum Field Theory (QFT)

Graduate-level overview – concise, organized, and ready for study

1. What is Quantum Field Theory?

Aspect	Classical Physics	Quantum Mechanics	Quantum Field Theory
Degrees of freedom	Finite number of particles with well-defined positions and momenta	Wavefunctions for a fixed number of particles	Fields $\phi(x)$ defined at every spacetime point (infinitely many degrees of freedom)
Fundamental objects	Particles, trajectories	State vectors $ \psi\rangle$	$ \psi\rangle$ in Hilbert space
Symmetries	Global (e.g., translation, rotation)	Global unitary transformations	Local (gauge) symmetries + global spacetime symmetries
Interactions	Forces as potentials $V(\mathbf{r})$	Potentials in Schrödinger equation	Interaction terms in the Lagrangian density $\mathcal{L}_{\text{int}}$

Key idea: Particles are excitations of underlying quantum fields. QFT unifies special relativity with quantum mechanics and provides the framework for the Standard Model of particle physics.

2. Classical Field Theory Refresher

- Field definition** – A scalar field $\phi(x)$ assigns a number to each spacetime point $x^\mu = (t, \mathbf{x})$. Vector, spinor, and gauge fields are defined analogously.
- Action principle** –
[$S[\phi] = \int d^4x; \mathcal{L}(\phi, \partial_\mu \phi)$] Stationary action ($\delta S = 0$) yields the Euler-Lagrange equations: [$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0$.]

Example: Real Klein-Gordon field

[$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m^2 \phi^2$

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$\Box + m^2 \phi = 0$

3. Canonical Quantization of Fields

Step	Classical	Quantum
Variables	$(\phi(\mathbf{x}, t), \pi(\mathbf{x}, t) = \partial \mathcal{L} / \partial \dot{\phi})$	Promote to operators
Commutation relations	Poisson brackets $(\phi, \pi) = \delta^3(\mathbf{x} - \mathbf{y})$	Equal-time commutators $[\hat{\phi}(\mathbf{x}, t), \hat{\pi}(\mathbf{y}, t)] = i\hbar \delta^3(\mathbf{x} - \mathbf{y})$ $[\hat{\phi}(\mathbf{x}, t), \hat{\phi}(\mathbf{y}, t)] = 0$ $[\hat{\pi}(\mathbf{x}, t), \hat{\pi}(\mathbf{y}, t)] = 0$
Mode expansion	Fourier series of solutions	Introduce creation/annihilation operators $(a_{\mathbf{k}}, a_{\mathbf{k}}^\dagger)$
Hamiltonian	$(H = \int d^3x \mathcal{H})$	Operator $(\hat{H})$ built from $a, a^\dagger$ – counts quanta