

Introduction to Special Relativity

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Key concepts, historical background, and fundamental results

1. Historical Context

Year	Milestone	Significance
1887	Michelson-Morley experiment	Failed to detect Earth's motion through the "luminiferous ether"; suggested that the speed of light is the same in all directions.
1905	Albert Einstein's paper " <i>On the Electrodynamics of Moving Bodies</i> "	Formulated the two postulates of special relativity, eliminating the need for an ether and reconciling Maxwell's equations with mechanics.
1905	Hermann Minkowski's spacetime formulation (1908)	Unified space and time into a four-dimensional continuum, providing a geometric interpretation of relativity.

2. Fundamental Postulates

1. Principle of Relativity

- The laws of physics are identical in all inertial (non-accelerating) reference frames.
- No inertial frame is physically privileged.

2. Constancy of the Speed of Light

- In vacuum, light propagates at a constant speed $c = 299,792,458 \text{ m/s}^1$, independent of the motion of the source or the observer.
- This speed is the same in every inertial frame.

3. Core Concepts

3.1. Inertial Frames & Coordinates

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- An *inertial frame* is one in which a free particle moves with constant velocity (Newton's first law holds).
- Coordinates are usually denoted $((t, x, y, z))$ in one frame (S) and $((t', x', y', z'))$ in another frame (S') moving at velocity $\mathbf{v}$ relative to S.

3.2. Lorentz Transformation

For motion along the x-axis:

$$\begin{aligned} t' &= \gamma \left(t - \frac{v x}{c^2} \right) & x' &= \gamma (x - v t) \\ y &= y' & z &= z' \end{aligned}$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (\text{Lorentz factor})$$

- Reduces to the Galilean transformation when $(v \ll c)$ ($\gamma \approx 1$).
- Guarantees that the spacetime interval remains invariant.

3.3. Spacetime Interval (Invariant)

$$\Delta s^2 = c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

- **Timelike** ($\Delta s^2 > 0$): events can be causally connected; a frame exists where they occur at the same spatial point.
- **Spacelike** ($\Delta s^2 < 0$): no causal influence; there is a frame where they are simultaneous.
- **Lightlike** ($\Delta s^2 = 0$): separation exactly on the light cone; describes the path of a photon.

3.4. Time Dilation

A moving clock runs slower as seen from a stationary frame:

$$\Delta t = \gamma \Delta \tau$$

- $\Delta \tau$ = proper time (time measured by a clock at rest relative to the event).
- Example: muons produced in the upper atmosphere (proper