

Introduction to Celestial Mechanics

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The study of how bodies move under the influence of gravity.

1. Fundamental Concepts

Concept	Description
Celestial Body	Any natural object in space (planet, moon, asteroid, comet, star, etc.).
Gravitational Force	Attractive force between any two masses; described by Newton's law of universal gravitation.
Orbit	The path a body follows around a central mass due to gravity.
Reference Frame	Coordinate system used to describe motion (inertial vs. rotating).
Two-Body Problem	Simplified case where only two masses interact; yields closed-form solutions (elliptic, parabolic, hyperbolic trajectories).
N-Body Problem	Realistic situation with three or more interacting bodies; generally solved numerically.

2. Newton's Law of Universal Gravitation

$$\mathbf{F}_{12} = -G \frac{m_1 m_2}{r^2} \hat{\mathbf{r}}_{12}$$

- G – gravitational constant ($6.674 \times 10^{-11} \text{ N} \cdot \text{m}^2 \cdot \text{kg}^{-2}$).
- m_1, m_2 – masses of the two bodies.
- r – distance between their centers.
- $\hat{\mathbf{r}}_{12}$ – unit vector from body 1 to body 2.

From this law, the **acceleration** of each body follows Newton's second law:

$$\mathbf{a}_1 = -G \frac{m_2}{r^2} \hat{\mathbf{r}}_{12}, \quad \mathbf{a}_2 = -G \frac{m_1}{r^2} \hat{\mathbf{r}}_{21}$$

3. Kepler's Laws (Result of the Two-Body Solution)

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1. **Law of Orbits** – Planets move in ellipses with the Sun at one focus.
2. **Law of Areas** – A line joining a planet to the Sun sweeps equal areas in equal times (conservation of angular momentum).
3. **Law of Periods** – The square of a planet's orbital period (T) is proportional to the cube of its semi-major axis (a):

$$[T^2 = \frac{4\pi^2}{G(M_{\odot} + m)} a^3]$$

For ($m \ll M_{\odot}$), the denominator reduces to ($GM_{\odot}$).

4. Orbital Elements (Six Parameters that Define an Orbit)

Element	Symbol	Meaning
Semi-major axis	(a)	Size of the orbit (average distance from the primary).
Eccentricity	(e)	Shape (0 = circle, $0 < e < 1$ = ellipse).
Inclination	(i)	Tilt of the orbital plane relative to a reference plane (e.g., ecliptic).
Longitude of Ascending Node	(Ω)	Angle from reference direction to the point where the orbit crosses the reference plane upward.
Argument of Periapsis	(ω)	Angle from the ascending node to the periapsis within the orbital plane.