

Introduction to Complex Numbers

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Key ideas, definitions, operations, and visual tools

1. What Is a Complex Number?

Symbol	Meaning
$\mathbb{R}$	Set of real numbers
i	Imaginary unit, defined by $i^2 = -1$
$\mathbb{C}$	Set of all numbers of the form $a + bi$ with $a, b \in \mathbb{R}$
Real part: $\operatorname{Re}(z) = a$ Imaginary part: $\operatorname{Im}(z) = b$	

A complex number is written $(z = a + bi)$.

If $(b = 0)$ the number is purely real; if $(a = 0)$ it is *purely imaginary*.

2. Geometric Representation

2.1 Argand (Complex) Plane

- Horizontal axis ? real part (a)
- Vertical axis ? imaginary part (b)

A point $((a,b))$ corresponds to the complex number $(a+bi)$.

2.2 Polar Form

$$[z = r(\cos\theta + i\sin\theta) = re^{i\theta}]$$

- Modulus (magnitude):** $(r = |z| = \sqrt{a^2 + b^2})$
- Argument (angle):** $(\theta = \arg(z) = \tan^{-1}\left(\frac{b}{a}\right))$ (adjusted for quadrant)

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3. Basic Operations

Operation	Algebraic Form	Polar Form
Addition	$((a+bi)+(c+di) = (a+c) + (b+d)i)$	Add vectors tip to tail in the plane
Subtraction	$((a+bi)-(c+di) = (a-c) + (b-d)i)$	Same as addition with the opposite vector
Multiplication	$((a+bi)(c+di) = (ac-bd) + (ad+bc)i)$	$(r_1 r_2, e^{i(\theta_1 + \theta_2)})$
Division	$(\frac{a+bi}{c+di} = \frac{(a+bi)(c-di)}{(c+di)(c-di)} = \frac{(ac+bd) + (bc-ad)i}{c^2+d^2})$	$(\frac{r_1}{r_2}, e^{i(\theta_1 - \theta_2)})$
Conjugate	$(\overline{z} = a-bi)$	$(\overline{z} = re^{-i\theta})$
Modulus	$(z = \sqrt{a^2+b^2})$	(r)
Argument	$(\arg(z) = \theta) \text{ (principal value } (-\pi < \theta \leq \pi))$	Same

Why the conjugate helps:

$$[z \overline{z} = (a+bi)(a-bi) = a^2 + b^2 = |z|^2]$$

4. Important Properties

1. **Field structure** – $(\mathbb{C})$ satisfies all field axioms (addition, multiplication, inverses).
2. **Algebraic closure** – Every non