

Group Actions

Group Actions

1. Definition

- A group action is a way a group G "acts" on a set X , formally a function $G \times X \rightarrow X$ satisfying $ex = x$ and $(gh)x = g(hx)$ for all g, h in G , x in X .

2. Orbit

- The orbit of an element x is the set of all elements it can be mapped to by applying every element of G .

3. Stabilizer

- The stabilizer of x is the subgroup of elements in G that leave x unchanged (fix x in place).

4. Orbit-Stabilizer Theorem

- The size of the orbit of x equals the index of the stabilizer of x in G ($|G| / |\text{stabilizer}|$) for finite groups.

5. Examples

- A symmetry group acting on the vertices of a polygon; a permutation group acting on a set of objects.

6. Why Group Actions Matter

- They provide a powerful tool for counting symmetric structures (via Burnside's lemma) and appear throughout physics, chemistry, and combinatorics.