Linear Transformations and Matrices

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1. **Definition of a Linear Transformation**
   - A function T: V -> W between vector spaces that preserves vector addition and scalar multiplication: T(u+v) = T(u)+T(v) and T(cv) = cT(v).
2. **Matrix Representation**
   - Every linear transformation between finite-dimensional vector spaces can be represented by a matrix once bases are chosen for V and W.
3. **Kernel and Image**
   - The **kernel** is the set of vectors mapped to zero; the **image** is the set of all possible outputs - both are subspaces.
4. **Rank-Nullity Theorem**
   - dim(kernel) + dim(image) = dim(V), linking the transformation's "loss of information" to the dimension of the domain.
5. **Composition of Transformations**
   - Corresponds to matrix multiplication: if S and T are linear transformations, their composition's matrix is the product of their individual matrices.
6. **Applications**
   - Linear transformations model rotations, scaling, and projections in computer graphics, and are foundational to data transformations in machine learning.