Vector Spaces

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1. **Definition**
   - A vector space over a field F is a set V with vector addition and scalar multiplication satisfying axioms like closure, associativity, distributivity, and existence of a zero vector.
2. **Key Axioms**
   - Vectors can be added together and scaled by field elements, always remaining within the vector space (closure).
3. **Subspaces**
   - A subset of a vector space that is itself a vector space under the same operations; must contain the zero vector and be closed under addition and scalar multiplication.
4. **Linear Independence**
   - A set of vectors is linearly independent if no vector in the set can be written as a combination of the others.
5. **Basis and Dimension**
   - A basis is a linearly independent set that spans the entire vector space; the number of vectors in a basis defines the space's dimension.
6. **Why Vector Spaces Matter**
   - They provide the abstract foundation for linear algebra, with direct applications in computer graphics, machine learning, physics, and engineering.