Rings and Fields

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1. **Definition of a Ring**
    - A set R with two operations (addition and multiplication) where R is an abelian group under addition, multiplication is associative, and multiplication distributes over addition.
2. **Commutative Rings**
    - A ring where multiplication is also commutative (ab = ba for all a, b in R).
3. **Rings with Unity**
    - A ring that has a multiplicative identity element (usually denoted 1).
4. **Definition of a Field**
    - A commutative ring with unity in which every non-zero element has a multiplicative inverse - essentially, a structure where addition, subtraction, multiplication, and division (except by zero) all work.
5. **Examples**
    - The integers form a ring (but not a field, since most integers lack multiplicative inverses within the integers). The rational, real, and complex numbers are all fields.
6. **Why Fields Matter**
    - Fields provide the algebraic setting for linear algebra, coding theory, and cryptography (e.g., finite fields underpin much of modern encryption).