Group Homomorphisms and Isomorphisms

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1. **Homomorphism Definition**
   - A function f: G -> H between two groups that preserves the group operation: f(a*b) = f(a)*f(b) for all a, b in G.
2. **Kernel of a Homomorphism**
   - The set of elements in G that map to the identity element in H; the kernel is always a subgroup of G (in fact, a normal subgroup).
3. **Isomorphism**
   - A homomorphism that is also bijective (one-to-one and onto), meaning the two groups have identical structure, just with relabeled elements.
4. **Why Isomorphisms Matter**
   - If two groups are isomorphic, any structural property true of one is true of the other - they are "the same group" in all algebraic respects.
5. **Image of a Homomorphism**
   - The set of outputs f(G) is always a subgroup of H.
6. **First Isomorphism Theorem**
   - States that G/ker(f) is isomorphic to the image of f, connecting homomorphisms, quotient groups, and isomorphisms.