

Subgroups and Lagrange's Theorem

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1. Definition of a Subgroup

- A subset H of a group G that is itself a group under the same operation as G .

2. Subgroup Test

- A non-empty subset H of G is a subgroup if it is closed under the operation and under taking inverses.

3. Trivial and Improper Subgroups

- Every group has at least two subgroups: the trivial subgroup $\{e\}$ and the group G itself.

4. Lagrange's Theorem

- For a finite group G , the order (size) of any subgroup H divides the order of G exactly.

5. Consequences of Lagrange's Theorem

- The order of any element in a finite group must divide the order of the group, which severely restricts what subgroups can exist.

6. Cosets

- Lagrange's theorem is proved by partitioning G into equal-sized cosets of H , showing $|G| = |H| \times (\text{number of cosets})$.