Introduction to Groups

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1. **Definition**
    - A group is a set G together with a binary operation * satisfying four axioms: closure, associativity, identity, and inverses.
2. **The Four Axioms**
    - **Closure**: for all a, b in G, a*b is also in G.
    - **Associativity**: (a*b)*c = a*(b*c) for all a, b, c in G.
    - **Identity**: there exists e in G such that a*e = e*a = a for all a.
    - **Inverses**: for every a in G, there exists a^-1 in G such that a*a^-1 = e.
3. **Examples of Groups**
    - Integers under addition, non-zero rational numbers under multiplication, and the set of symmetries of a shape under composition.
4. **Abelian Groups**
    - A group where the operation is also commutative (a*b = b*a for all a, b); named after mathematician Niels Henrik Abel.
5. **Order of a Group**
    - The number of elements in a group (finite groups) or the fact that it is infinite (infinite groups).
6. **Why Groups Matter**
    - Groups formalize the idea of symmetry and structure, with applications in cryptography, physics, and chemistry alongside pure mathematics.