

Introduction to Differential Equations

Introduction to Differential Equations

Compact, structured notes for quick reference and deeper study

1. What Is a Differential Equation (DE)?

Aspect	Description
Definition	An equation that relates a function $y(x)$ (or several functions) to one or more of its derivatives.
Goal	Find the unknown function(s) that satisfy the relation.
Notation	Typically written as $F(x, y, y', y'', \dots, y^{(n)}) = 0$.
Order	Highest derivative appearing (e.g., first order, second order).
Degree	Power of the highest derivative when the equation is polynomial in derivatives (only defined for algebraic DEs).
Linear vs. Non-linear	Linear: each term is either a function of x or a derivative multiplied by a function of x ; no products or powers of the unknown function or its derivatives. Non-linear: any other case.

2. Classification of DEs

Category	Sub-type	Typical Form	Example
Ordinary vs. Partial	ODE – one independent variable	$y' = f(x, y)$	$y' = xy$
	PDE – several independent variables	$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$	Heat equation
Order	First order, second order, ...	$y^{(n)} = f(x, y, y', \dots, y^{(n-1)})$	$y'' + 3y' = \sin x$
Linearity	Linear	$a_n(x) y^{(n)} + \dots + a_0(x) y = g(x)$	$y'' - 4y' + 4y = e^x$
	Non-linear	Any term like y^2 , (yy') , $((y')^2)$	$y' = y^2$

Introduction to Differential Equations

Category	Sub?type	Typical Form	Example
Homogeneous vs. Non?homogeneous	Homogeneous (no free term)	$(L[y]=0)$	$(y''+y=0)$
	Non?homogeneous	$(L[y]=g(x))$	$(y''+y = \cos x)$
Exactness	Exact (total differential)	$(M(x,y)dx+N(x,y)dy=0)$ with $(\partial M/\partial y = \partial N/\partial x)$	$((2xy + y^2)dx + (x^2 + 2xy)dy = 0)$
	Non?exact (requires integrating factor)	—	—

3. First?Order Differential Equations

3.1 Separable Equations

Form: $\frac{dy}{dx}=g(x)h(y)$

Solution steps

1. Rewrite: $\frac{1}{h(y)}dy = g(x)dx$.
2. Integrate both sides: $\int \frac{1}{h(y)}dy = \int g(x)dx + C$.

Example

$\frac{dy}{dx}=xy \Rightarrow \frac{1}{y}dy = xdx \Rightarrow \ln|y| = \frac{x^2}{2} + C$