

Introduction to Vectors

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Key concepts, definitions, and operations for a solid foundation in vector mathematics.

1. What Is a Vector?

Aspect	Description
Definition	A quantity that has both magnitude (size) and direction .
Notation	Usually boldface $\mathbf{v}$, an arrow ($\vec{v}$), or a letter with a hat for a unit vector ($\hat{v}$).
Examples	Displacement, velocity, acceleration, force, electric field.
Contrast with Scalars	Scalars have only magnitude (e.g., mass, temperature, time).

2. Representing Vectors

2.1 Geometric (Arrow) Form

- Draw an arrow from point **A** (tail) to point **B** (head).
- Length $\rightarrow$ magnitude, orientation $\rightarrow$ direction.

2.2 Component (Algebraic) Form

- In a Cartesian coordinate system (2D or 3D):
2D: $\vec{v} = \langle v_x, v_y \rangle$
3D: $\vec{v} = \langle v_x, v_y, v_z \rangle$
- Components are projections of the vector onto the coordinate axes.

2.3 Unit Vectors

- A unit vector has magnitude 1 and indicates direction.
- Standard basis vectors: $\hat{i} = \langle 1, 0, 0 \rangle$, $\hat{j} = \langle 0, 1, 0 \rangle$, $\hat{k} = \langle 0, 0, 1 \rangle$
- Any vector can be expressed as a linear combination: $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

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3. Vector Magnitude (Length)

For a vector $(\vec{v} = \langle v_x, v_y, v_z \rangle)$:

$$[|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}]$$

- In 2D, drop the (v_z) term.
 - The magnitude is always non-negative.
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4. Vector Operations

Operation	Symbol	Formula (3D)	Geometric Meaning
Addition	$(\vec{u} + \vec{v})$	$(\langle u_x + v_x, u_y + v_y, u_z + v_z \rangle)$	“Tip-to-tail” (parallelogram)
Subtraction	$(\vec{u} - \vec{v})$	$(\langle u_x - v_x, u_y - v_y, u_z - v_z \rangle)$	Vector from the head of $\vec{v}$ to the head of $\vec{u}$
Scalar Multiplication	$(c\vec{v})$	$(\langle c v_x, c v_y, c v_z \rangle)$	Stretches/compresses magnitude; reverses direction if $(c < 0)$
Dot (Scalar) Product	$(\vec{u} \cdot \vec{v})$	$(u_x v_x + u_y v_y + u_z v_z = \quad \backslash$	