

Introduction to Limits

Introduction to Limits

Calculus I – Core Concept

1. What Is a Limit?

A **limit** describes the behavior of a function $f(x)$ as the input x approaches a particular value c .

Formally:

$$\lim_{x \rightarrow c} f(x) = L$$

means that **the values of $f(x)$ can be made arbitrarily close to L by taking x sufficiently close (but not equal) to c .**

- L may be a finite number, $+\infty$, $-\infty$, or may not exist.
- The definition does **not** require $f(c)$ to be defined; limits capture the trend, not the actual point.

2. Why Limits Matter

Application	How Limits Are Used
Derivative	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$
Integral (Riemann sum)	$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \Delta x$
Continuity	f is continuous at c iff $\lim_{x \rightarrow c} f(x) = f(c)$
Asymptotes & behavior at infinity	$\lim_{x \rightarrow \infty} f(x)$ tells us horizontal asymptotes.
Series convergence	$\sum_{n=1}^{\infty} a_n$ converges when $\lim_{n \rightarrow \infty} s_n$ (partial sums) exists.

3. Intuitive Picture

Introduction to Limits

- **Approaching from the left** ($x \rightarrow c^-$): values of $f(x)$ as x comes from smaller numbers.
- **Approaching from the right** ($x \rightarrow c^+$): values of $f(x)$ as x comes from larger numbers.

If both one-sided limits exist and are equal, the two-sided limit exists.

4. Formal (ϵ - δ) Definition

For a finite limit L :

$$\lim_{x \rightarrow c} f(x) = L \iff \text{for all } \epsilon > 0, \exists \delta > 0 \text{ such that } 0 < |x - c| < \delta \implies |f(x) - L| < \epsilon.$$

- **(ϵ)** (epsilon) = desired closeness of $f(x)$ to L .
- **(δ)** (delta) = required closeness of x to c .

Key idea: No matter how tight a tolerance we set for the output, we can find a tolerance for the input that guarantees it.

5. Common Types of Limits

Type	Typical Form	How to Evaluate
Finite limit at a finite point	$\lim_{x \rightarrow c} f(x)$	Direct substitution (if continuous), factor & cancel, rationalize, use trigonometric identities.
Infinite limit	$\lim_{x \rightarrow c} f(x)$	