

# Introduction to Geometry

## Introduction to Geometry – Study Notes

### 1. What Is Geometry?

- Definition:** Geometry is the branch of mathematics that deals with the properties, measurement, and relationships of points, lines, surfaces, and solids.
- Origins:** The word comes from the Greek *geo* (earth) + *metron* (measure). Early geometry was developed for land surveying, astronomy, and architecture.
- Main Goal:** To describe and understand the shape, size, and relative position of objects in space.

### 2. Fundamental Concepts

| Concept | Description                                                | Typical Notation              |
|---------|------------------------------------------------------------|-------------------------------|
| Point   | Location with no size or dimension.                        | (A, B, P)                     |
| Line    | Extends infinitely in two directions; one-dimensional.     | ( $\overleftrightarrow{AB}$ ) |
| Ray     | Starts at a point and extends infinitely in one direction. | ( $\overrightarrow{AB}$ )     |
| Segment | Part of a line bounded by two endpoints.                   | ( $\overline{AB}$ )           |
| Plane   | Flat, two-dimensional surface extending infinitely.        | ( $\Pi$ )                     |
| Angle   | Measure of rotation between two rays sharing a vertex.     | ( $\angle ABC$ )              |
| Polygon | Closed figure formed by straight line segments.            | Triangle, quadrilateral, etc. |
| Solid   | Three-dimensional object (e.g., cube, sphere).             | —                             |

### 3. Types of Geometry

| Branch        | Focus                                     | Example Topics                           |
|---------------|-------------------------------------------|------------------------------------------|
| Euclidean     | Flat space; based on Euclid's postulates. | Parallel lines, triangle congruence.     |
| Non-Euclidean | Curved spaces (hyperbolic, spherical).    | Spherical triangles, hyperbolic tilings. |

# Introduction to Geometry

| Branch           | Focus                                                   | Example Topics                          |
|------------------|---------------------------------------------------------|-----------------------------------------|
| Analytic         | Uses coordinates & algebra to study geometric objects.  | Distance formula, equations of circles. |
| Transformational | Studies motions (translations, rotations, reflections). | Symmetry, isometries.                   |
| Differential     | Uses calculus to examine curves & surfaces.             | Curvature, surface area via integrals.  |

---

## 4. Euclidean Postulates (Brief)

1. A straight line can be drawn between any two points.
2. A finite line segment can be extended indefinitely.
3. A circle can be drawn with any center and radius.
4. All right angles are congruent.
5. **Parallel Postulate** – Through a point not on a given line, exactly one line can be drawn parallel to the given line.

These axioms form the logical foundation for most elementary geometry.

---

## 5. Key Relationships & Theorems

### 5.1 Triangle Basics

- **Sum of interior angles:**  $(180^\circ)$ .
- **Pythagorean theorem** (right triangle):  $(a^2+b^2=c^2)$ .
- **Congruence criteria:** SSS, SAS, ASA, AAS, HL (right triangles).
- **Similarity criteria:** AA, SSS, SAS (proportional sides).

### 5.2 Quadrilaterals

| Quadrilateral | Properties                                              |
|---------------|---------------------------------------------------------|
| Parallelogram | Opposite sides parallel & equal; opposite angles equal. |
| Rectangle     | Parallelogram with all right angles.                    |
| Rhombus       | Parallelogram with all sides equal.                     |

# Introduction to Geometry

**Quadrilateral**

**Properties**

**Square**

Both rectangle and rhombus; all sides equal