

Conduction in Composite Materials

Conduction in Composite Materials – Structured Study Notes

1. Overview

Composite materials are engineered by combining two or more distinct phases (matrix + reinforcement) to obtain properties that cannot be achieved by the constituents alone. Heat conduction in composites is governed by the thermal conductivities of the individual phases, their geometry, orientation, and the quality of the interface.

2. Fundamental Concepts

Concept	Definition	Relevance to Composites
Thermal Conductivity (k)	Measure of a material's ability to conduct heat ($\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$).	Each phase (fiber, filler, matrix) has its own k; the overall k of the composite is a combination of these values.
Fourier's Law	$q = -k \frac{dT}{dx}$ (heat flux q is proportional to the temperature gradient).	Used to model heat flow in each constituent and across interfaces.
Effective Thermal Conductivity (k_eff)	The equivalent conductivity of the heterogeneous composite treated as a homogeneous medium.	Design target for thermal management applications.
Anisotropy	Direction-dependent properties.	Fiber-reinforced composites often exhibit different k_{parallel} and $k_{\text{perpendicular}}$ to the fiber direction.
Thermal Contact Resistance (TCR)	Additional resistance at the interface due to imperfect bonding, surface roughness, or interphase layers.	Can dominate the overall resistance in high-performance composites.

3. Micromechanical Models for k_eff

3.1 Rule of Mixtures (Series & Parallel)

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Configuration	Equation	When to Use
Parallel (isostrain)	$(k_{\text{eff}} = V_f k_f + V_m k_m)$	Fibers aligned with heat flow; high conductivity direction.
Series (isostress)	$(\frac{1}{k_{\text{eff}}} = \frac{V_f}{k_f} + \frac{V_m}{k_m})$	Heat flow perpendicular to fibers; low conductivity direction.

V_f = volume fraction of filler/fiber, V_m = matrix volume fraction ($V_f + V_m = 1$).

3.2 Maxwell-Eucken Model (Dilute Dispersions)

$$[k_{\text{eff}} = k_m \frac{k_f + 2k_m + 2V_f (k_f - k_m)}{k_f + 2k_m - V_f (k_f - k_m)}]$$

- Assumes spherical inclusions, low volume fraction ($V_f \leq 0.3$).
- Useful for polymer composites with ceramic particles.

3.3 Effective Medium Approximation (EMA)

$$[\sum_i V_i \frac{k_i - k_{\text{eff}}}{k_i + A k_{\text{eff}}} = 0]$$

- A is a geometry factor ($A = 2$ for spherical inclusions, $A = 0$ for aligned plates).
- Handles higher filler loadings and arbitrary shapes.

3.4 Mori-Tanaka Method (Aligned Fibers)

$$[k_{\parallel} = k_m \left(1 + \frac{V_f (k_f - k_m)}{k_m + \xi (k_f - k_m)} \right)]$$

$$[k_{\perp} = k_m \left(1 + \frac{V_f (k_f - k_m)}{k_f + \xi (k_m - k_f)} \right)]$$

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