

# Introduction to Optimization Techniques

## Introduction to Optimization Techniques

Course?level overview – concise, structured, and easy to read

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### 1. What Is Optimization?

| Aspect      | Description                                                                                        |
|-------------|----------------------------------------------------------------------------------------------------|
| Goal        | Find the “best” solution among all feasible alternatives.                                          |
| Best        | Defined by a <b>objective function</b> (e.g., minimize cost, maximize profit, reduce error).       |
| Feasibility | Solutions must satisfy a set of <b>constraints</b> (equalities, inequalities, logical conditions). |
| Domain      | Variables may be continuous, discrete, integer, binary, or a mix.                                  |

#### Typical formulation

$$\begin{aligned} &\text{minimize / maximize} \quad f(\mathbf{x}) \\ &\text{subject to} \\ &\quad g_i(\mathbf{x}) \leq 0, \; i=1, \dots, m \quad \& \quad h_j(\mathbf{x}) = 0, \; j=1, \dots, p \\ &\quad \mathbf{x} \in \mathcal{X} \end{aligned}$$

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### 2. Classification of Optimization Problems

| Class                                       | Variable Type               | Objective  | Typical Methods                                    |
|---------------------------------------------|-----------------------------|------------|----------------------------------------------------|
| Linear Programming (LP)                     | Continuous                  | Linear     | Simplex, interior?point                            |
| Integer Programming (IP)                    | Integer / binary            | Linear     | Branch?and?bound, cutting planes                   |
| Mixed?Integer Linear Programming (MILP)     | Mix of continuous & integer | Linear     | Branch?and?cut, Benders                            |
| Nonlinear Programming (NLP)                 | Continuous                  | Non?linear | Gradient?based (SQP, interior?point), trust?region |
| Mixed?Integer Nonlinear Programming (MINLP) | Mix                         | Non?linear | Outer?approximation, ECP, heuristic hybrids        |

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| Class                            | Variable Type           | Objective                             | Typical Methods                                                     |
|----------------------------------|-------------------------|---------------------------------------|---------------------------------------------------------------------|
| Convex Optimization              | Continuous              | Convex objective & convex constraints | Interior-point, proximal methods                                    |
| Combinatorial Optimization       | Discrete (graphs, sets) | Often linear                          | Dynamic programming, greedy, approximation algorithms               |
| Stochastic / Robust Optimization | Continuous or discrete  | Expectation or worst-case             | Sample-average approximation, scenario trees                        |
| Metaheuristics                   | Any                     | Black-box                             | Genetic algorithms, simulated annealing, particle swarm, ant colony |

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## 3. Core Concepts & Terminology

| Term                       | Meaning                                                                                       |
|----------------------------|-----------------------------------------------------------------------------------------------|
| Feasible region            | Set of all $\mathbf{x}$ that satisfy constraints.                                             |
| Optimal solution           | Feasible point that yields the best objective value.                                          |
| Local vs. global optimum   | Local: best in a neighbourhood; Global: best overall.                                         |
| Convexity                  | If feasible region and objective are convex ? any local optimum is global.                    |
| Duality                    | Every primal problem has a dual; dual solutions give bounds and sensitivity information.      |
| Sensitivity / Shadow price | Change in optimal value per unit change in a constraint RHS.                                  |
| KKT conditions             | Necessary (and under convexity, sufficient) conditions for optimality in NLP.                 |
| Relaxation                 | Removing/integrating constraints (e.g., integer ? continuous) to obtain a bound.              |
| Branch-and-bound           | Systematically explore subproblems, pruning by bounds.                                        |
| Cutting plane              | Add linear constraints that cut off infeasible integer points without removing feasible ones. |

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## 4. Solution Approaches

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## 4.1 Exact Algorithms

Technique When to use Key idea

Simplex