

Introduction to Electromagnetic Fields

Introduction to Electromagnetic Fields

Graduate-level overview – concise, organized, and ready for quick reference.

1. Fundamental Concepts

Concept	Definition	Symbol	Units
Electric Field	Region of space where a charge experiences a force	$\mathbf{E}$	$\text{V} \cdot \text{m}^{-1}$ ($\text{N} \cdot \text{C}^{-1}$)
Magnetic Field	Region where a moving charge (or current) experiences a force	$\mathbf{B}$	tesla (T) = $\text{N} \cdot \text{A}^{-1} \cdot \text{m}^{-1}$
Electromagnetic Field	Unified description of $\mathbf{E}$ and $\mathbf{B}$ that obey Maxwell's equations	—	—
Field Line	Imaginary line tangent to the field vector at every point; density $\propto$ magnitude	—	—

2. Maxwell's Equations (Differential Form)

Equation	Physical Meaning	Mathematical Form
Gauss's Law (Electric)	Electric flux through a closed surface equals enclosed charge	$\nabla \cdot \mathbf{E} = \rho / \epsilon_0$
Gauss's Law (Magnetic)	No magnetic monopoles; net magnetic flux = 0	$\nabla \cdot \mathbf{B} = 0$
Faraday's Law	Time-varying magnetic field induces an electric field	$\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t$
Ampère-Maxwell Law	Currents and changing electric fields produce magnetic fields	$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \partial \mathbf{E} / \partial t$

ϵ_0 – vacuum permittivity ($8.854 \times 10^{-12} \text{ F} \cdot \text{m}^{-1}$)

μ_0 – vacuum permeability ($4\pi \times 10^{-7} \text{ H} \cdot \text{m}^{-1}$)

ρ – charge density, $\mathbf{J}$ – current density.

Introduction to Electromagnetic Fields

3. Wave Propagation

3.1 Derivation (source-free region)

Starting from curl equations with $\rho = 0$, $\mathbf{J} = 0$:

1. Take curl of Faraday's law $\nabla \times (\nabla \times \mathbf{E}) = -\frac{\partial}{\partial t} (\nabla \times \mathbf{B})$
2. Use vector identity $\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$ reduces to $-\nabla^2 \mathbf{E}$ (since $\nabla \cdot \mathbf{E} = 0$).
3. Substitute Ampère-Maxwell: $\nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$.

Result: $\mathbf{E}$ satisfies the wave equation

$$\boxed{\nabla^2 \mathbf{E} - \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0}$$

A similar equation holds for $\mathbf{B}$.

3.2 Wave speed

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \approx 2.998 \times 10^8 \text{ m}\cdot\text{s}^{-1}$$

4. Energy and Momentum

Quantity	Expression	Interpretation
Energy density	$u = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \mu_0 H^2$	