

Introduction to Calculus

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Key ideas, fundamental concepts, and why calculus matters

1. What Is Calculus?

Aspect	Description
Definition	The branch of mathematics that studies change (differentiation) and accumulation (integration).
Two Main Branches	Differential Calculus – focuses on rates of change and slopes of curves. Integral Calculus – focuses on areas, volumes, and the accumulation of quantities.
Core Principle	The Fundamental Theorem of Calculus links differentiation and integration, showing they are inverse processes.

2. Why Study Calculus?

- Model Real-World Phenomena** – motion, growth, decay, economics, biology, engineering, etc.
- Provide Tools for Higher Mathematics** – analysis, differential equations, multivariable calculus.
- Develop Logical Thinking** – rigorous proof techniques and problem-solving strategies.

3. Fundamental Concepts

3.1 Limits

- Idea:** The value a function approaches as the input gets arbitrarily close to a point.
- Notation:** $\lim_{x \rightarrow a} f(x) = L$
- Key Uses:** Defining continuity, derivatives, and integrals.

3.2 Continuity

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- A function f is **continuous** at $(x=a)$ if
$$\lim_{x \rightarrow a} f(x) = f(a).$$
- Continuous functions have no “jumps” or “holes.”

3.3 Derivative (Differential Calculus)

- **Geometric meaning:** Slope of the tangent line to the curve $(y=f(x))$ at a point.
- **Definition:**
$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$
- **Notation options:** $(f'(x),; \frac{dy}{dx},; D_x f)$

Basic Rules

Rule	Formula
Power	$\frac{d}{dx} x^n = nx^{n-1}$
Constant multiple	$\frac{d}{dx} [c, f(x)] = c, f'(x)$
Sum/Difference	$\frac{d}{dx} [f \pm g] = f' \pm g'$
Product	$(fg)' = f'g + fg'$
Quotient	$\left(\frac{f}{g} \right)' = \frac{f'g - fg'}{g^2}$
Chain	$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

3.4 Integral (Integral Calculus)

- **Geometric meaning:** Area under the curve $(y=f(x))$ between two points.
- **Indefinite Integral (Antiderivative):**
$$\int f(x), dx = F(x) + C$$
 where $(F'(x)=f(x)).$
- **Definite Integral:**
$$\int_a^b f(x), dx$$
 gives the signed area from $(x=a)$ to $(x=b).$

Basic Rules

Rule	Formula
Power $(n \neq -1)$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C$